UB-ECM-PF 06/28

The nucleon-nucleon potential beyond the static approximation

Jorge Mondejar, Joan Soto

Deptartament d'Estructura i Constituents de la Matèria, Universitat de Barcelona

Diagonal 647, 08028 Barcelona, Catalonia, Spain

Abstract

We point out that, due to the use of static nucleon propagators in Heavy Baryon Chiral Perturbation Theory, the current calculations of the nucleon-nucleon potential miss certain contributions starting at two loops. These contributions give rise to contact interactions, which are both parametrically and numerically more important than the so called NNLO potentials. They show a peculiar dependence on the light quark masses, which should be taken into account when performing chiral extrapolations of lattice data.

I. INTRODUCTION

Since Weinberg's pioneering work in the early 90's [1, 2], there has been an enormous developement of effective theory methods for few nucleon systems (see [3–5] for recent reviews). However, there is still a lack of consensus on how calculations in the so called NN effective theory including pions must be organized [6–12][13–16][17, 18][19–25][26–30][31, 32][33–36][37][38][39]. Starting from the Heavy Baryon Chiral Lagrangian (HBχL)[40], one is interested in building a low energy effective theory for nucleon energies much lower than the pion mass. It seems clear that the remaining effective theory must consist of nucleons interacting through a potential. Hence the program may be divided in two: (i) calculating the potential and (ii) organising the remaining quantum mechanical calculation. We shall not enter in point (ii) here, which is where the main difficulties arise. Concerning point (i), following the so called Weinberg approach, it is commonly believed that one can use HBχPT counting rules and the outcome of the calculation can be easily organized in standard chiral counting in $1/\Lambda_\chi$; $\Lambda_\chi \sim 4\pi F_\pi$, F_π being the pion decay constant. We point out here that this is not so. If we understand the potentials as matching coefficients which arise after integrating out higher energy degrees of freedom [39, 41], then, starting at two loops, there are contributions to them which are missed if the static approximation is used for the nucleon propagators, as prescribed by HBχPT counting rules [52]. This is due to the fact that the energy scale given by the pion mass m_π has an associated three-momentum scale $\sqrt{m_\pi M_N}$, M_N being the nucleon mass, which may make the kinetic term of the nucleon propagator as important as the energy when integrating out degrees of freedom of energy m_π [53]. This becomes apparent when individual Feynman diagrams are analyzed with the method of the strategy of regions or threshold expansions [45, 46] with non-relativistic nucleon propagators, as will be shown in the next section. This kind of contributions are related to the so called radiation pions discussed in Refs. [15, 16] within the KSW approach[13, 14], but, to our knowledge, they have never been discussed in the framework of the original Weinberg proposal for calculating the potential[1, 2]. We present in section III their complete contribution at two loops. In order to obtain it, the leading order lagrangian (in the isopin limit) augmented by the kinetic terms of the nucleon suffices,

$$\mathcal{L} = \mathcal{L}_\pi + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} \tag{1}$$

The purely pionic sector reads

$$\mathcal{L}_\pi = \frac{F_\pi^2}{4}\left\{Tr[\nabla_\mu \mathrm{U}^\dagger \nabla^\mu \mathrm{U} + \mathrm{m}_\pi^2(\mathrm{U}+\mathrm{U}^\dagger)]\right\}\ ,\ U = e^{i\frac{\pi^a\tau^a}{F_\pi}}\ , \tag{2}$$

π^a is the pion field and τ^a the (isospin) Pauli matrices. The pion-nucleon sector reads

$$\mathcal{L}_{\pi N} = N^\dagger \left(iD_0 - g_A(\mathbf{u}\cdot\frac{\boldsymbol{\sigma}}{2}) + \frac{\mathbf{D}^2}{2M_N}\right) N\ . \tag{3}$$

N is the nucleon field, g_A is the axial vector coupling constant of the nucleon, $\boldsymbol{\sigma}$ the (spin) Pauli matrices, and defining $u^2(x) = U(x)$ the covariant derivative of the nucleon field is given by

$$D_\mu N = \left(\partial_\mu + \frac{1}{2}[u^\dagger, \partial_\mu u]\right) N\ , \tag{4}$$

and the axial-vector type object u_μ by,

$$u_\mu = i(u^\dagger \nabla_\mu u - u\nabla_\mu u^\dagger) = i\{u^\dagger, \nabla_\mu u\} = iu^\dagger \nabla_\mu U u^\dagger\ . \tag{5}$$

Finally, the nucleon-nucleon sector reads

$$\mathcal{L}_{NN} = -\frac{C_S}{2} N^\dagger N N^\dagger N - \frac{C_T}{2} N^\dagger \boldsymbol{\sigma} N N^\dagger \boldsymbol{\sigma} N \tag{6}$$

where C_S and C_T are low energy constants to be determined from the experiment or from QCD, for instance by lattice simulations [47, 48]. In section IV we discuss the relevance of this region for higher order contributions and the significance of our results. Section V is devoted to the conclusions. Details of our calculation are presented in the Appendix.

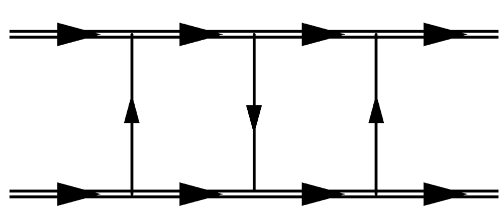
II. THRESHOLD EXPANSIONS: A SAMPLE CALCULATION

In this section we will introduce the method of calculation that we will apply. Many processes in physics, like those involving heavy quarks or our pion-nucleon interaction, involve more than one mass scale. Such processes are notoriously difficult to calculate in perturbation theory beyond the one-loop level. To proceed one has to resort to approximations, either numerical or analytical. Among the latter finds its place the strategy of regions [45, 46]. The idea is to perform an asymptotic expansion of the integrals in certain ratios of mass scales, so that the resulting integrals appearing in the calculation of every term in the expansion are simpler, and the expansion is homogeneous (each integral appearing in the construction contributes to a determinate power in the expansion parameter). In short, the method goes as follows:

1. Determine the large and small scales in the problem.
2. Introduce factorization scales μ_i and divide the loop integration domain into regions in which the loop momentum is of the order of one of the scales in the problem.
3. Perform, in every region, a Taylor expansion in the parameters which are small in the given region, and stay at leading order. At this point, keep only the relevant regions and discard the rest: the relevant regions are those that somehow maintain the structure of poles of the original integral. If we integrate over multiple momenta, and the integrand has several propagators, so that we have one or more poles associated with each momentum, after performing the Taylor expansion we should still have at least one pole for each momentum. If we end up loosing all the poles that were associated with one of the variables of integration, the region we are considering is irrelevant and must not be taken into account.
4. After expansion, ignore all factorization scales and integrate over the entire loop integration domain in every relevant region.

The non-trivial point to justify is 4, which also guarantees the homogeneity of the expansion formula. In order for that point to hold it is essential to use dimensional (or analytic) regularization for the integral, even if it is finite in four dimensions. Loosely speaking, 4 follows in dimensional regularization from the property that all integrals without scale vanish, but the truth is that at present day there are no mathematical proofs of the method of regions. The best we can say is that it has not failed yet, giving asymptotic expansions for any diagram in any limit and having been checked in numerous examples when comparing results of expansion with existing explicit analytical results.

We will now show how this method works and how it enables us to find new contributions to the nucleon-nucleon potential. Consider the following diagram,

I = 

$$
\begin{aligned}
= i\left(\frac{g_A}{F_\pi}\right)^6 (7\vec{\tau}_1\cdot\vec{\tau}_2-6)\frac{\sigma_1^i}{2}\frac{\sigma_1^j}{2}\frac{\sigma_1^l}{2}\frac{\sigma_2^r}{2}\frac{\sigma_2^s}{2}\frac{\sigma_2^t}{2}\int\frac{d^dk}{(2\pi)^d}\int\frac{d^dl}{(2\pi)^d}k^lk^tl^jl^s(k-l+p-p')^i(k-l+p-p')^r \\
\times\frac{1}{(k-l+\vec{p}-\vec{p}')^2-m_\pi^2+i\epsilon}\quad\frac{1}{l^2-m_\pi^2+i\epsilon}\quad\frac{1}{k^2-m_\pi^2+i\epsilon}\quad\frac{1}{k^0+p^0-\frac{(\vec{k}+\vec{p})^2}{2M_N}+i\epsilon} \\
\times\frac{1}{k^0-l^0+p^0-\frac{(\vec{k}-\vec{l}+\vec{p})^2}{2M_N}+i\epsilon}\quad\frac{1}{k^0-p^0+\frac{(\vec{k}+\vec{p})^2}{2M_N}-i\epsilon}\quad\frac{1}{k^0-l^0-p^0+\frac{(\vec{k}-\vec{l}+\vec{p})^2}{2M_N}-i\epsilon}\,,
\end{aligned}
\tag{7}
$$

where we put ourselves in the center of mass frame, that is,

$$
\begin{array}{ccc} p_1=(p^0,\vec{p}) & & p_3=(p^0,\vec{p}') \\ & , & \\ p_2=(p^0,-\vec{p}) & & p_4=(p^0,-\vec{p}') \end{array}\;. \tag{8}
$$

We will assume that the nucleon momentum $\mathbf{p}$ and $\mathbf{p}'$, and the momentum transfer $\mathbf{p}-\mathbf{p}'$ are of order m_π whereas the nucleon energy p_0 is of order m_π^2/M_N, which fixes the scales of the effective theory. The relevant energy scales in the diagram are m_π^2/M_N and m_π and the associated three momentum scales m_π and $\sqrt{m_\pi M_N}$ respectively. We will have in mind the philosophy described in [39]: modes with energies larger than m_π^2/M_N are integrated out giving rise to the potential. The strategy of regions will help us to separate these modes from the ones which must be kept in the effective theory (see [49] for an example of such a calculation). Now we must break the integration domain into pieces. The relevant regions are easily found by inspecting the poles of the propagators: they are the parts of the integration domain in which one or more of the propagators inside the integral develop a pole. Let us display next the leading term of the Taylor expansion of the integrand in several regions:

- $$
\left\{\begin{array}{l} k^0\sim m_\pi\ ,\ \vec{k}\sim m_\pi \\ \\ l^0\sim m_\pi\ ,\ \ \vec{l}\sim m_\pi \end{array}\right.\qquad k^0-l^0\sim m_\pi \tag{9}
$$

$$
\frac{(k-l+p-p')^i(k-l+p-p')^r}{(k-l+\vec{p}-\vec{p}')^2-m_\pi^2+i\epsilon}\quad\frac{l^jl^s}{l^2-m_\pi^2+i\epsilon}\quad\frac{k^lk^t}{k^2-m_\pi^2+i\epsilon} \tag{10}
$$

$$\times \frac{1}{k^0+i\epsilon} \quad \frac{1}{k^0-l^0+i\epsilon} \quad \frac{1}{k^0-i\epsilon}\frac{1}{k^0-l^0-i\epsilon}\ .$$

This is a pure three pion exchange potential contribution, the same we find in the static approximation.

•

$$\begin{cases} k^0 \sim m_\pi\ ,\ \vec{k} \sim m_\pi \\ & k^0 - l^0 \sim \frac{m_\pi^2}{M_N} \\ l^0 \sim m_\pi\ ,\ \ \vec{l} \sim m_\pi \end{cases} \tag{11}$$

$$\frac{(k-l+p-p')^i(k-l+p-p')^r}{-(\vec{k}-\vec{l}+\vec{p}-\vec{p}')^2-m_\pi^2} \quad \frac{l^j l^s}{l^2-m_\pi^2+i\epsilon} \quad \frac{k^l k^t}{k^2-m_\pi^2+i\epsilon} \tag{12}$$
$$\times \frac{1}{k^0+i\epsilon} \quad \frac{1}{k^0-i\epsilon} \quad \frac{1}{k^0-l^0+p^0-\frac{(\vec{k}-\vec{l}+\vec{p})^2}{2M_N}+i\epsilon} \quad \frac{1}{k^0-p^0+\frac{(\vec{k}+\vec{p})^2}{2M_N}-i\epsilon}\ .$$

We recognize here the contribution of a two pion exchange potential followed by a one pion exchange potential, a contribution that also exists in the effective theory (i.e. in the quantum mechanical calculation) and hence, to avoid double counting, must be discarded.

•

$$\begin{cases} k^0 \sim \frac{m_\pi^2}{M_N}\ ,\ \vec{k} \sim m_\pi \\ \\ l^0 \sim m_\pi\ ,\ \ \vec{l} \sim m_\pi \end{cases} \tag{13}$$

$$\frac{(k-l+p-p')^i(k-l+p-p')^r}{(\vec{k}-l+\vec{p}-\vec{p}')^2-m_\pi^2+i\epsilon} \quad \frac{l^j l^s}{l^2-m_\pi^2+i\epsilon} \quad \frac{k^l k^t}{-\vec{k}^2-m_\pi^2} \tag{14}$$
$$\times \frac{1}{k^0+p^0-\frac{(\vec{k}+\vec{p})^2}{2M_N}+i\epsilon} \quad \frac{1}{k^0-p^0+\frac{(\vec{k}+\vec{p})^2}{2M_N}-i\epsilon} \quad \frac{1}{l^0-i\epsilon} \quad \frac{1}{l^0+i\epsilon}\ .$$

As opposed to the former region, this one gives a one pion potential followed by a two pion potential, and must be discarded likewise.

•

$$\begin{cases} k^0 \sim \frac{m_\pi^2}{M_N}\ ,\ \vec{k} \sim m_\pi \\ \\ l^0 \sim \frac{m_\pi^2}{M_N}\ ,\ \ \vec{l} \sim m_\pi \end{cases} \tag{15}$$

$$\frac{(k-l+p-p')^i(k-l+p-p')^r}{-(\vec{k}-\vec{l}+\vec{p}-\vec{p}')^2-m_\pi^2} \quad \frac{l^j l^s}{-\vec{l}^2-m_\pi^2} \quad \frac{k^l k^t}{-\vec{k}^2-m_\pi^2} \quad \frac{1}{k^0+p^0-\frac{(\vec{k}+\vec{p})^2}{2M_N}+i\epsilon}$$
$$\times\frac{1}{k^0-p^0+\frac{(\vec{k}+\vec{p})^2}{2M_N}-i\epsilon} \quad \frac{1}{k^0-l^0+p^0-\frac{(\vec{k}-\vec{l}+\vec{p})^2}{2M_N}+i\epsilon} \quad \frac{1}{k^0-l^0-p^0+\frac{(\vec{k}-\vec{l}+\vec{p})^2}{2M_N}-i\epsilon} \ . \tag{16}$$

This is a one pion potential iterated three times, which also exists in the effective theory, and hence is to be dropped like the other contributions.

-
$$\begin{cases} k^0 \sim m_\pi \ , \ \vec{k} \sim \sqrt{M_N m_\pi} \\ \\ l^0 \sim m_\pi \ , \ \vec{l} \sim \sqrt{M_N m_\pi} \end{cases} \quad k^0-l^0 \sim m_\pi \ , \ \vec{k}-\vec{l} \sim \sqrt{M_N m_\pi} \tag{17}$$

$$\frac{(k-l)^i(k-l)^r}{-(\vec{k}-\vec{l})^2} \quad \frac{l^j l^s}{-\vec{l}^2} \quad \frac{k^l k^t}{-\vec{k}^2} \quad \frac{1}{k^0-\frac{\vec{k}^2}{2M_N}+i\epsilon} \quad \frac{1}{k^0+\frac{\vec{k}^2}{2M_N}-i\epsilon} \tag{18}$$
$$\times\frac{1}{k^0-l^0-\frac{(\vec{k}-\vec{l})^2}{2M_N}+i\epsilon} \quad \frac{1}{k^0-l^0+\frac{(\vec{k}-\vec{l})^2}{2M_N}-i\epsilon} \ .$$

After performing the integrations over k^0 and l^0 we are left with

$$\frac{(k-l)^i(k-l)^r}{-(\vec{k}-\vec{l})^2} \quad \frac{l^j l^s}{-\vec{l}^2} \quad \frac{k^l k^t}{-\vec{k}^2} \quad M_N \frac{1}{\vec{k}^2\left((\vec{k}-\vec{l})^2-\vec{k}^2\right)} \ . \tag{19}$$

The remaining integrand has no scales, and therefore vanishes when we integrate over the remaining $d-1$ dimensions for k and l using dimensional regularization.

By analogous arguments, we find that all the remaining regions have also vanishing contributions, except for the following one.

-
$$\begin{cases} k^0 \sim m_\pi \ , \ \vec{k} \sim \sqrt{M_N m_\pi} \\ \\ l^0 \sim m_\pi \ , \ \vec{l} \sim m_\pi \end{cases} \quad k^0-l^0 \sim m_\pi \tag{20}$$

$$\frac{k^i k^t}{-\vec{k}^2} \quad \frac{l^j l^s}{l^2-m_\pi^2+i\epsilon} \quad \frac{k^l k^t}{-\vec{k}^2} \quad \frac{1}{k^0-\frac{\vec{k}^2}{2M_N}+i\epsilon} \quad \frac{1}{k^0+\frac{\vec{k}^2}{2M_N}-i\epsilon} \tag{21}$$

$$\times \frac{1}{k^0 - l^0 - \frac{\vec{k}^2}{2M_N} + i\epsilon} \quad \frac{1}{k^0 - l^0 + \frac{\vec{k}^2}{2M_N} - i\epsilon} \ .$$

This contribution, that is not the composition of one or two pion potentials, nor does it vanish, is the new piece of the nucleon-nucleon potential arising from considering non-relativistic (rather than static) nucleon propagators.

So, summarizing, after the analysis of poles of the propagator we have found a purely three pion exchange potential (which is the result of the static case), various iterations of one and two pion exchange potentials, and a new contribution given by

$$\begin{aligned} I^{new} &= i\left(\frac{g_A}{F_\pi}\right)^6 (7\vec{\tau}_1\cdot\vec{\tau}_2 - 6)\frac{\sigma_1^i}{2}\frac{\sigma_1^j}{2}\frac{\sigma_1^l}{2}\frac{\sigma_2^r}{2}\frac{\sigma_2^s}{2}\frac{\sigma_2^t}{2}\int\frac{d^dk}{(2\pi)^d}\int\frac{d^dl}{(2\pi)^d}\frac{k^ik^r}{-\vec{k}^2}\quad\frac{l^jl^s}{l^2-m_\pi^2+i\epsilon}\quad\frac{k^lk^t}{-\vec{k}^2} \\ &\times\frac{1}{k^0-\frac{\vec{k}^2}{2M_N}+i\epsilon}\quad\frac{1}{k^0+\frac{\vec{k}^2}{2M_N}-i\epsilon}\quad\frac{1}{k^0-l^0-\frac{\vec{k}^2}{2M_N}+i\epsilon}\quad\frac{1}{k^0-l^0+\frac{\vec{k}^2}{2M_N}-i\epsilon} \ . \end{aligned} \tag{22}$$

The tensorial part of our integral reduces to

$$\begin{aligned} &\left(\frac{\sigma_1^i}{2}\frac{\sigma_1^j}{2}\frac{\sigma_1^i}{2}\frac{\sigma_2^r}{2}\frac{\sigma_2^j}{2}\frac{\sigma_2^r}{2}+\frac{\sigma_1^i}{2}\frac{\sigma_1^j}{2}\frac{\sigma_1^l}{2}\frac{\sigma_2^i}{2}\frac{\sigma_2^j}{2}\frac{\sigma_2^l}{2}+\frac{\sigma_1^i}{2}\frac{\sigma_1^j}{2}\frac{\sigma_1^l}{2}\frac{\sigma_2^l}{2}\frac{\sigma_2^j}{2}\frac{\sigma_2^i}{2}\right)\frac{\vec{k}^4\vec{l}^2}{(d-1)^2(d+1)} \\ &\qquad = \left(\frac{2}{(d-1)(d+1)}+\frac{1}{d+1}\right)\frac{\vec{\sigma_1}\cdot\vec{\sigma_2}}{2^6}\vec{k}^4\vec{l}^2 \ . \end{aligned} \tag{23}$$

And so (22) becomes

$$\begin{aligned} I^{new} &= i\left(\frac{g_A}{F_\pi}\right)^6 (7\vec{\tau}_1\cdot\vec{\tau}_2-6)\left(\frac{2}{(d-1)(d+1)}+\frac{1}{d+1}\right)\frac{\vec{\sigma_1}\cdot\vec{\sigma_2}}{64}\int\frac{d^{d-1}\vec{k}}{(2\pi)^{d-1}}\int\frac{d^{d-1}\vec{l}}{(2\pi)^{d-1}}\vec{k}^4\vec{l}^2 \\ &\times\quad\frac{M_N^3}{\vec{k}^8}\quad\frac{1}{\sqrt{\vec{l}^2+m_\pi^2}}\quad\frac{1}{\vec{k}^2+M_N\sqrt{\vec{l}^2+m_\pi^2}} \ . \end{aligned} \tag{24}$$

Integrating now, we find that the new contribution is

$$\begin{aligned} I^{new} &= i\left(\frac{g_A}{F_\pi}\right)^6 (7\vec{\tau}_1\cdot\vec{\tau}_2-6)\left(\frac{2}{(d-1)(d+1)}+\frac{1}{d+1}\right)\frac{\vec{\sigma_1}\cdot\vec{\sigma_2}}{64}\frac{M_N^{\frac{d-1}{2}}m_\pi^{\frac{3d-7}{2}}}{2^{2d-2}\pi^{d-1}\left(\Gamma\left(\frac{d-1}{2}\right)\right)^2} \\ &\times\frac{\Gamma\left(\frac{d+1}{2}\right)\Gamma\left(\frac{7-3d}{4}\right)}{\Gamma\left(\frac{9-d}{4}\right)}\Gamma\left(\frac{d-5}{2}\right)\Gamma\left(\frac{7-d}{2}\right) \ . \end{aligned} \tag{25}$$

Regularizing in $d = 4 - 2\epsilon$ dimensions this is

$$\begin{aligned} I^{new} &= i\left(\frac{g_A}{F_\pi}\right)^6 (7\vec{\tau}_1\cdot\vec{\tau}_2-6)\frac{1}{3}\frac{\vec{\sigma_1}\cdot\vec{\sigma_2}}{64}\frac{M_N^{\frac{3}{2}}m_\pi^{\frac{5}{2}}}{64\pi^3\left(\Gamma\left(\frac{3}{2}\right)\right)^2}\frac{\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\Gamma\left(\frac{-1}{2}\right)\Gamma\left(\frac{3}{2}\right) \\ &= i\left(\frac{g_A}{F_\pi}\right)^6 (7\vec{\tau}_1\cdot\vec{\tau}_2-6)\frac{1}{3}\frac{\vec{\sigma_1}\cdot\vec{\sigma_2}}{64}\frac{M_N^{\frac{3}{2}}m_\pi^{\frac{5}{2}}}{16\pi^4}\frac{\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\frac{3\sqrt{\pi}}{4}(-\pi) \ . \end{aligned} \tag{26}$$

III. RESULTS

By repeating the analysis of the previous section, one can show that only the diagrams displayed in the Appendix produce extra contributions when non-relativistic propagators are used instead of static ones. The contribution of some of the diagrams containing two local vertices (from (6)) can be extracted from [50]. In particular we have checked that the ratio of the pieces proportional to fractional powers of the masses arising from the diagrams Fig. 1 b) and Fig. 1 f) in that reference agrees with ratio of (A.13) and (A.16) in the Appendix, as it should. There are further diagrams involving two pion vertices with contributions in this region, but they are suppressed by powers of m_π/M_N with respect to the ones displayed in the Appendix. This is due to the fact that two pion vertices go with time derivatives ($\sim m_\pi$) rather than with space ones ($\sim \sqrt{m_\pi M_N}$). Adding up all remaining contributions, and using the identities

$$\begin{aligned} N^\dagger \tau^a \boldsymbol{\sigma} N N^\dagger \tau^a \boldsymbol{\sigma} N &= -3 N^\dagger N N^\dagger N \\ N^\dagger \tau^a N N^\dagger \tau^a N &= -2 N^\dagger N N^\dagger N - N^\dagger \boldsymbol{\sigma} N N^\dagger \boldsymbol{\sigma} N \end{aligned} \tag{27}$$

we obtain the following contribution to the (momentum space) potential

$$V = \frac{3 M_N^{\frac{3}{2}} m_\pi^{\frac{5}{2}} \Gamma\left(\frac{-5}{4}\right)}{128 \pi^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right)} \left\{ \frac{1}{2}\left(\frac{g_A}{F_\pi}\right)^6 + \left(\frac{g_A}{F_\pi}\right)^2 \left[C_S^2 - C_S C_T \left(2(\vec{\sigma}_1\cdot\vec{\sigma}_2)+4\right) + C_T^2 \left(2(\vec{\sigma}_1\cdot\vec{\sigma}_2)+23\right)\right] \right\} \tag{28}$$

This amounts to the following redefinition of C_S and C_T

$$\begin{aligned} C_S \longrightarrow \tilde{C_S} &= C_S + \frac{3 M_N^{\frac{3}{2}} m_\pi^{\frac{5}{2}} \Gamma\left(\frac{-5}{4}\right)}{128 \pi^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right)} \left\{ \frac{1}{2}\left(\frac{g_A}{F_\pi}\right)^6 + \left(\frac{g_A}{F_\pi}\right)^2 \left(C_S^2 - 4 C_S C_T + 23 C_T^2 \right) \right\} \\ &\equiv C_S + \Delta C_S \end{aligned} \tag{29}$$

$$\begin{aligned} C_T \longrightarrow \tilde{C_T} &= C_T + \frac{3 M_N^{\frac{3}{2}} m_\pi^{\frac{5}{2}} \Gamma\left(\frac{-5}{4}\right)}{128 \pi^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right)} \left\{ 2\left(\frac{g_A}{F_\pi}\right)^2 \left(-C_S C_T + C_T^2 \right) \right\} \\ &\equiv C_T + \Delta C_T \end{aligned} \tag{30}$$

Hence the previous extractions from data of these coefficients must be corrected according to the formula above. For instance, we may use the extraction found in ref. [7]. There we may find different results for C_S and C_T in the np channel coming from a fit done for different values of the cut-offs Λ and $\tilde{\Lambda}$, which enter the Lippmann-Schwinger equation and the spectral-function representation of the two-pion exchange potential, respectively. The approximate results are shown in Table I.

LEC	$\{450, 500\}$	$\{600, 600\}$	$\{450, 700\}$	$\{600, 700\}$
C_S	-10.7	8.9	-12.1	3.4
C_T	-1.2	5.3	-0.6	2.5

TABLE I: The approximate values of the S-wave LECs C_S and C_T in the np channel at N^3LO for the different cut-off combinations $\{\Lambda[\mathrm{MeV}], \tilde{\Lambda}[\mathrm{MeV}]\}$, from the fit in [7]. The values of the constants are in 10^{-5} MeV^{-2}.

If we use these values of C_S and C_T to find an estimate of the size of the corrections ΔC_S and ΔC_T, we find the values displayed in Table II. Clearly these are no small corrections.

Δ	$\{450, 500\}$	$\{600, 600\}$	$\{450, 700\}$	$\{600, 700\}$
ΔC_S	21.2	54.1	23.3	23.5
ΔC_T	-1.6	-2.8	-1.0	-0.3

TABLE II: The values of the corrections ΔC_S and ΔC_T calculated using C_S and C_T from the former table as input. We use $g_A = 1.29$, $F_\pi = 92.4$ MeV, $M_N = 939$ MeV, and $m_\pi = 139$ MeV. The values of the constants are in 10^{-5} MeV^{-2}.

In fact, if using the data from [7] we solve equations (29) and (30) exactly, we find complex values for C_S and C_T in all cases. This would indicate the need to redo the fits in [7] taking into account these new contributions.

IV. DISCUSSION

At first sight the new contributions to the potential we have found may look irrelevant since they amount to redefinitions of local counterterms. This is probably so as far as the description of scattering data is concerned. However, they may be of practical importance at least in the following two issues: (i) they shift the values of C_S and C_T extracted from data, which is important in order to check the consistency of a given counting scheme, and (ii) they indicate that there will be contributions going like $m_q^{5/4}$, m_q being the light quark masses, which should be taken into account if one aims at a precision calculation of the nucleon-nucleon scattering lengths from the lattice QCD using chiral extrapolations [51].

In higher loop calculations, subdiagrams with nucleon energies scaling as m_π and nucleon three-momentum as $\sqrt{m_\pi M_N}$ will appear. It is not difficult to convince oneself that such subdiagrams are produced by adding an extra one pion exchange or a contact term to the two loop diagrams we have calculated. These additions amount to a "suppression" of a factor $M_N^{3/2} m_\pi^{1/2}/\Lambda_\chi^2 \sim 1.2$. Hence all these higher loop calculations should better be summed up, which might provide an explanation to the unnatural size of the scattering lengths. Whether this is feasible or not will be left for future work.

Let us finally point out that this kind of contributions put into question the parametric applicability of the KSW power counting at energies $\sim m_\pi$, independently of its known problems of convergence when confronted with data at NNLO [16]. KSW argued that, in order to take into account the large scattering lengths of the nucleon-nucleon system, the four nucleon contact terms are enhanced and must be counted as $1/(\Lambda_\chi m_\pi)$ rather than as $1/\Lambda_\chi^2$ [13]. Since $M_N \sim \Lambda_\chi$, these contributions would produce a divergent expansion. In Ref. [50], where contributions of these kind were identified within the KSW framework, the prescription was taken to use the analytic continuation of the sum of the series.

V. CONCLUSIONS

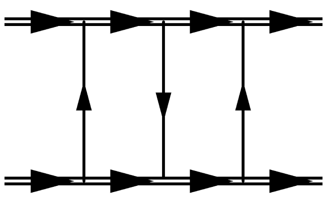
We have found contributions to the nucleon-nucleon potential which are missed if static rather than non-relativistic nucleon propagators are used in the calculation and have calculated the leading contributions of them, which appear at two loops. They produce large contact terms with a peculiar non-analytic dependence on the light quark masses.

Acknowledgements.

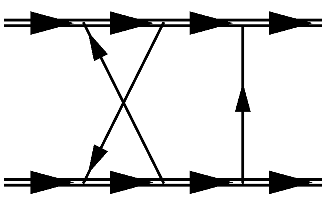
JS thanks Harald Griesshammer and Daniel Phillips for important discussions. He is also thankful to the organizers of ECT* activity *Effective Theories in Nuclear Physics and Lattice QCD*, where these discussions took place and part of this work was carried out. JM is supported by a *Distinció* from the *Generalitat de Catalunya*. We acknowledge financial support from MEC (Spain) grant CYT FPA 2004-04582-C02-01, the CIRIT (Catalonia) grant 2005SGR00564, and the RTNs Euridice HPRN-CT2002-00311 and Flavianet MRTN-CT-2006-035482 (EU).

APPENDIX A: RESULTS FOR INDIVIDUAL DIAGRAMS

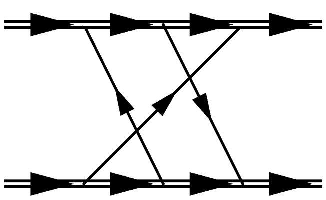
1. Diagrams with no contact terms



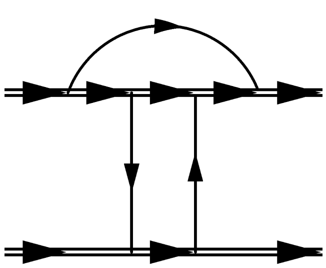
$$= i\left(\frac{g_A}{F_\pi}\right)^6 (7\vec{\tau}_1\cdot\vec{\tau}_2 - 6)\frac{1}{3}\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{64}\frac{M_N^{\frac{3}{2}}m_\pi^{\frac{5}{2}}}{16\pi^4}\frac{\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\frac{3\sqrt{\pi}}{4}(-\pi) \tag{A1}$$



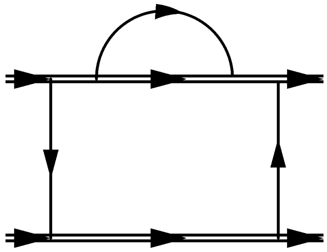
$$= 2i\left(\frac{g_A}{F_\pi}\right)^6 (6 - \vec{\tau}_1\cdot\vec{\tau}_2)\left(\frac{-1}{9}\right)\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{64}\frac{M_N^{\frac{3}{2}}m_\pi^{\frac{5}{2}}}{32\pi^4}\frac{\Gamma\left(-\frac{5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\frac{3\sqrt{\pi}}{4}\pi \tag{A2}$$



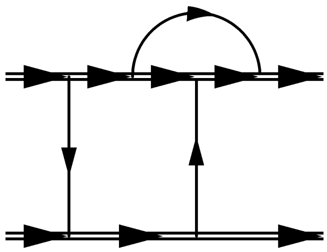
$$= i\left(\frac{g_A}{F_\pi}\right)^6 (-\vec{\tau}_1\cdot\vec{\tau}_2 - 6)\frac{1}{3}\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{64}\frac{M_N^{\frac{3}{2}}m_\pi^{\frac{5}{2}}}{32\pi^4}\frac{\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\frac{3\sqrt{\pi}}{4}(-\pi) \tag{A3}$$



$$= i\left(\frac{g_A}{F_\pi}\right)^6 (9 + 2\vec{\tau}_1\cdot\vec{\tau}_2)\,\frac{1}{64}\frac{M_N^{\frac{3}{2}}m_\pi^{\frac{5}{2}}}{32\pi^4}\frac{\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\frac{3\sqrt{\pi}}{4}(-\pi) \tag{A4}$$

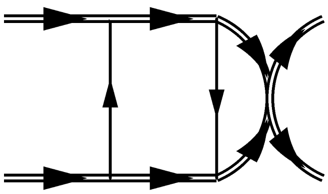


$$= i\left(\frac{g_A}{F_\pi}\right)^6 (9-6\vec{\tau}_1\cdot\vec{\tau}_2)\frac{1}{64}\frac{M_N^{\frac{3}{2}}m_\pi^{\frac{5}{2}}}{32\pi^4}\frac{\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\frac{3\sqrt{\pi}}{4}(-\pi) \qquad \text{(A5)}$$

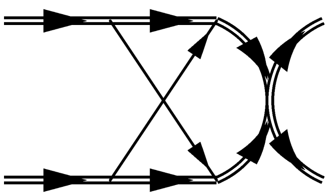


$$= i\left(\frac{g_A}{F_\pi}\right)^6 (2\vec{\tau}_1\cdot\vec{\tau}_2-3)\left(\frac{-1}{3}\right)\frac{1}{64}\frac{M_N^{\frac{3}{2}}m_\pi^{\frac{5}{2}}}{32\pi^4}\frac{\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\frac{3\sqrt{\pi}}{4}\pi \qquad \text{(A6)}$$

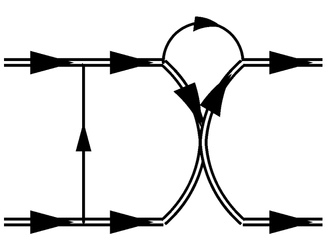
2. Diagrams with one contact term



$$= \frac{i}{2!}\left(\frac{g_A}{F_\pi}\right)^4 \frac{2}{9}(3-2\vec{\tau}_1\cdot\vec{\tau}_2)\left(-C_S(3-2\vec{\sigma}_1\cdot\vec{\sigma}_2)-C_T(7\vec{\sigma}_1\cdot\vec{\sigma}_2-6)\right)\frac{1}{16}$$
$$\times\frac{M_N^{\frac{3}{2}}m_\pi^{\frac{5}{2}}}{16\pi^4}\frac{\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\frac{3\sqrt{\pi}}{4}(-\pi) \qquad \text{(A7)}$$

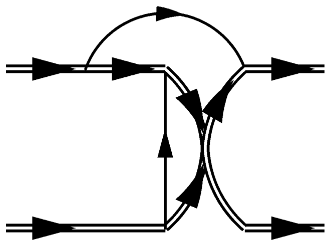


$$= 2\frac{i}{2!}\left(\frac{g_A}{F_\pi}\right)^4 \frac{2}{9}(3+2\vec{\tau}_1\cdot\vec{\tau}_2)(-C_S(3+2\vec{\sigma}_1\cdot\vec{\sigma}_2)-C_T(6-\vec{\sigma}_1\cdot\vec{\sigma}_2))\frac{1}{16}$$
$$\times\frac{M_N^{\frac{3}{2}}m_\pi^{\frac{5}{2}}}{32\pi^4}\frac{\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\frac{3\sqrt{\pi}}{4}\pi \qquad \text{(A8)}$$



$$= i\left(\frac{g_A}{F_\pi}\right)^4 \frac{1}{9}\left[(3\vec{\tau}_1\cdot\vec{\tau}_2)\left(-3C_S(\vec{\sigma}_1\cdot\vec{\sigma}_2)-C_T(2\vec{\sigma}_1\cdot\vec{\sigma}_2-3)\right)\right.$$
$$\left.+(3-2\vec{\tau}_1\cdot\vec{\tau}_2)\left(-C_S(3-2\vec{\sigma}_1\cdot\vec{\sigma}_2)-C_T(6-\vec{\sigma}_1\cdot\vec{\sigma}_2)\right)\right]$$

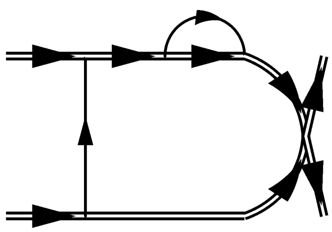
$$\times \frac{1}{16} \frac{M_N^{\frac{3}{2}} m_\pi^{\frac{5}{2}}}{32\pi^4} \frac{\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)} \frac{3\sqrt{\pi}}{4} \pi \tag{A9}$$



$$= i \left(\frac{g_A}{F_\pi}\right)^4 \frac{1}{9} \left[\left(-\vec{\tau}_1 \cdot \vec{\tau}_2\right) \left(C_S(\vec{\sigma}_1 \cdot \vec{\sigma}_2) - C_T(9 + 2\vec{\sigma}_1 \cdot \vec{\sigma}_2)\right) \right.$$

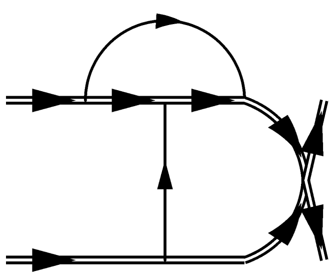
$$\left. + (3 + 2\vec{\tau}_1 \cdot \vec{\tau}_2) \left(-C_S(3 + 2\vec{\sigma}_1 \cdot \vec{\sigma}_2) + C_T(6 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)\right) \right] \frac{1}{16}$$

$$\times \frac{M_N^{\frac{3}{2}} m_\pi^{\frac{5}{2}}}{32\pi^4} \frac{\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)} \frac{3\sqrt{\pi}}{4} (-\pi) \tag{A10}$$



$$= \frac{i}{2!} \left(\frac{g_A}{F_\pi}\right)^4 \frac{2}{9} (3\vec{\tau}_1 \cdot \vec{\tau}_2) 3 (-C_S(\vec{\sigma}_1 \cdot \vec{\sigma}_2) - C_T(3 - 2\vec{\sigma}_1 \cdot \vec{\sigma}_2)) \frac{1}{16}$$

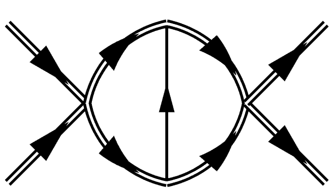
$$\times \frac{M_N^{\frac{3}{2}} m_\pi^{\frac{5}{2}}}{32\pi^4} \frac{\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)} \frac{3\sqrt{\pi}}{4} (-\pi) \tag{A11}$$



$$= \frac{i}{2!} \left(\frac{g_A}{F_\pi}\right)^4 \frac{2}{9} (-\vec{\tau}_1 \cdot \vec{\tau}_2) \left(C_S \vec{\sigma}_1 \cdot \vec{\sigma}_2 - C_T(-3 + 2\vec{\sigma}_1 \cdot \vec{\sigma}_2)\right) \frac{1}{16}$$

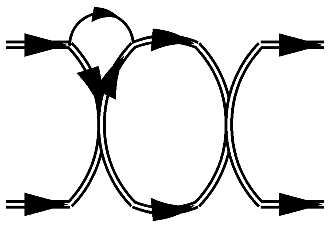
$$\times \frac{M_N^{\frac{3}{2}} m_\pi^{\frac{5}{2}}}{32\pi^4} \frac{\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)} \frac{3\sqrt{\pi}}{4} \pi \tag{A12}$$

3. Diagrams with two contact terms



$$= \frac{i}{(2!)^2} \left(\frac{g_A}{F_\pi}\right)^2 \frac{4}{3} (\vec{\tau}_1 \cdot \vec{\tau}_2) \left(C_S^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2 + 2C_S C_T(3 - 2\vec{\sigma}_1 \cdot \vec{\sigma}_2)\right.$$

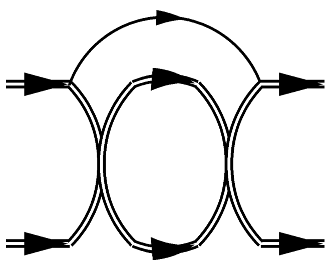
$$+C_T^2(7\vec{\sigma}_1\cdot\vec{\sigma}_2-6)\Big)\,\frac{1}{4}\frac{M_N^{\frac{3}{2}}m_\pi^{\frac{5}{2}}}{16\pi^4}\frac{\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\frac{3\sqrt{\pi}}{4}(-\pi) \tag{A13}$$



$$= \frac{i}{2!}\left(\frac{g_A}{F_\pi}\right)^2\frac{2}{3}\left[3(3C_S^2+2C_SC_T(\vec{\sigma}_1\cdot\vec{\sigma}_2)-C_T^2(3-2\vec{\sigma}_1\cdot\vec{\sigma}_2))\right.$$

$$\left.+(\vec{\tau}_1\cdot\vec{\tau}_2)(C_S^2(\vec{\sigma}_1\cdot\vec{\sigma}_2)+6C_SC_T+C_T^2(6-\vec{\sigma}_1\cdot\vec{\sigma}_2))\right]$$

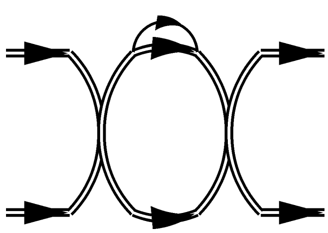
$$\times\frac{1}{4}\frac{M_N^{\frac{3}{2}}m_\pi^{\frac{5}{2}}}{32\pi^4}\frac{\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\frac{3\sqrt{\pi}}{4}\pi \tag{A14}$$



$$= i\left(\frac{g_A}{F_\pi}\right)^2\frac{2}{3}\left[3(3C_S^2-2C_SC_T(\vec{\sigma}_1\cdot\vec{\sigma}_2)+C_T^2(9+2\vec{\sigma}_1\cdot\vec{\sigma}_2))\right.$$

$$\left.+(\vec{\tau}_1\cdot\vec{\tau}_2)\left(C_S^2(\vec{\sigma}_1\cdot\vec{\sigma}_2)+2C_SC_T(3+2\vec{\sigma}_1\cdot\vec{\sigma}_2)-C_T^2(6+\vec{\sigma}_1\cdot\vec{\sigma}_2)\right)\right]\frac{1}{4}$$

$$\times\frac{M_N^{\frac{3}{2}}m_\pi^{\frac{5}{2}}}{32\pi^4}\frac{\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\frac{3\sqrt{\pi}}{4}(-\pi) \tag{A15}$$



$$= \frac{i}{(2!)^2}\left(\frac{g_A}{F_\pi}\right)^2\frac{4}{3}9(C_S^2+2C_SC_T(\vec{\sigma}_1\cdot\vec{\sigma}_2)+C_T^2(3-2\vec{\sigma}_1\vec{\sigma}_2))\frac{1}{4}$$

$$\times\frac{M_N^{\frac{3}{2}}m_\pi^{\frac{5}{2}}}{32\pi^4}\frac{\Gamma\left(\frac{-5}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\frac{3\sqrt{\pi}}{4}(-\pi) \tag{A16}$$

[1] S. Weinberg, Phys. Lett. B **251**, 288 (1990).

[2] S. Weinberg, Nucl. Phys. B **363**, 3 (1991).

[3] P. F. Bedaque and U. van Kolck, Ann. Rev. Nucl. Part. Sci. **52**, 339 (2002) [arXiv:nucl-th/0203055].

[4] E. Epelbaum, Prog. Part. Nucl. Phys. **57**, 654 (2006) [arXiv:nucl-th/0509032].

[5] H. W. Hammer, N. Kalantar-Nayestanaki and D. R. Phillips, arXiv:nucl-th/0611084.

[6] E. Epelbaum and U. G. Meissner, arXiv:nucl-th/0609037.

[7] E. Epelbaum, W. Glockle and U. G. Meissner, Nucl. Phys. A **747**, 362 (2005) [arXiv:nucl-th/0405048].

[8] E. Epelbaum, W. Gloeckle and U. G. Meissner, Nucl. Phys. A **671**, 295 (2000) [arXiv:nucl-th/9910064].

[9] E. Epelbaum, W. Gloeckle and U. G. Meissner, Nucl. Phys. A **637**, 107 (1998) [arXiv:nucl-th/9801064].

[10] C. Ordonez and U. van Kolck, Phys. Lett. B **291**, 459 (1992).

[11] C. Ordonez, L. Ray and U. van Kolck, Phys. Rev. Lett. **72**, 1982 (1994).

[12] C. Ordonez, L. Ray and U. van Kolck, Phys. Rev. C **53**, 2086 (1996) [arXiv:hep-ph/9511380].

[13] D. B. Kaplan, M. J. Savage and M. B. Wise, Phys. Lett. B **424**, 390 (1998) [arXiv:nucl-th/9801034].

[14] D. B. Kaplan, M. J. Savage and M. B. Wise, Nucl. Phys. B **534**, 329 (1998) [arXiv:nucl-th/9802075].

[15] S. Fleming, T. Mehen and I. W. Stewart, Phys. Rev. C **61**, 044005 (2000) [arXiv:nucl-th/9906056].

[16] S. Fleming, T. Mehen and I. W. Stewart, Nucl. Phys. A **677**, 313 (2000) [arXiv:nucl-th/9911001].

[17] A. Nogga, R. G. E. Timmermans and U. van Kolck, Phys. Rev. C **72**, 054006 (2005) [arXiv:nucl-th/0506005].

[18] S. R. Beane, P. F. Bedaque, M. J. Savage and U. van Kolck, Nucl. Phys. A **700**, 377 (2002) [arXiv:nucl-th/0104030].

[19] M. P. Valderrama and E. R. Arriola, Phys. Rev. C **72**, 044007 (2005).

[20] M. Pavon Valderrama and E. Ruiz Arriola, arXiv:nucl-th/0507075.

[21] M. P. Valderrama and E. R. Arriola, Phys. Rev. C **74**, 054001 (2006) [arXiv:nucl-th/0506047].

[22] M. Pavon Valderrama and E. Ruiz Arriola, Phys. Rev. C **72**, 054002 (2005) [arXiv:nucl-th/0504067].

[23] M. Pavon Valderrama and E. Ruiz Arriola, Phys. Rev. C **70**, 044006 (2004) [arXiv:nucl-th/0405057].

[24] M. Pavon Valderrama and E. Ruiz Arriola, Phys. Lett. B **580**, 149 (2004) [arXiv:nucl-th/0306069].

[25] J. Nieves, Phys. Lett. B **568**, 109 (2003) [arXiv:nucl-th/0301080].

[26] D. Djukanovic, J. Gegelia, S. Scherer and M. R. Schindler, arXiv:nucl-th/0609055.

[27] J. Gegelia and S. Scherer, Int. J. Mod. Phys. A **21**, 1079 (2006) [arXiv:nucl-th/0403052].

[28] J. Gegelia and G. Japaridze, Phys. Lett. B **517**, 476 (2001) [arXiv:nucl-th/0108005].

[29] J. Gegelia and G. Japaridze, Phys. Rev. D **60**, 114038 (1999) [arXiv:hep-ph/9908377].

[30] J. Gegelia, arXiv:nucl-th/9806028.

[31] V. S. Timoteo, T. Frederico, A. Delfino and L. Tomio, Phys. Lett. B **621**, 109 (2005) [arXiv:nucl-th/0508006].

[32] T. Frederico, V. S. Timoteo and L. Tomio, Nucl. Phys. A **653**, 209 (1999) [arXiv:nucl-th/9902052].

[33] J. F. Yang, J. H. Huang and D. Liu, Chin. Phys. Lett. **23**, 2688 (2006) [arXiv:nucl-th/0609068].

[34] J. F. Yang and J. H. Huang, arXiv:nucl-th/0410014.

[35] J. F. Yang and J. H. Huang, Phys. Rev. C **71**, 034001 (2005) [Erratum-ibid. C **71**, 069901 (2005)] [arXiv:nucl-th/0409023].

[36] J. F. Yang, arXiv:nucl-th/0407090.

[37] M. C. Birse, Phys. Rev. C **74**, 014003 (2006) [arXiv:nucl-th/0507077].

[38] J. A. Oller, Nucl. Phys. A **725**, 85 (2003).

[39] D. Eiras and J. Soto, Eur. Phys. J. A **17**, 89 (2003) [arXiv:nucl-th/0107009].

[40] E. Jenkins and A. V. Manohar, Phys. Lett. B **255**, 558 (1991).

[41] A. Pineda and J. Soto, Nucl. Phys. Proc. Suppl. **64**, 428 (1998) [arXiv:hep-ph/9707481].

[42] T. Becher and H. Leutwyler, Eur. Phys. J. C **9**, 643 (1999) [arXiv:hep-ph/9901384].

[43] D. Eiras and J. Soto, Phys. Rev. D **61**, 114027 (2000) [arXiv:hep-ph/9905543].

[44] N. Brambilla, A. Pineda, J. Soto and A. Vairo, Phys. Lett. B **580**, 60 (2004) [arXiv:hep-ph/0307159].

[45] M. Beneke and V. A. Smirnov, Nucl. Phys. B **522**, 321 (1998) [arXiv:hep-ph/9711391].

[46] V. A. Smirnov, Springer Tracts Mod. Phys. **177**, 1 (2002).

[47] M. Fukugita, Y. Kuramashi, M. Okawa, H. Mino and A. Ukawa, Phys. Rev. D **52**, 3003 (1995) [arXiv:hep-lat/9501024].

[48] S. R. Beane, P. F. Bedaque, K. Orginos and M. J. Savage, Phys. Rev. Lett. **97**, 012001 (2006) [arXiv:hep-lat/0602010].

[49] B. A. Kniehl, A. A. Penin, V. A. Smirnov and M. Steinhauser, Nucl. Phys. B **635**, 357 (2002) [arXiv:hep-ph/0203166].

[50] T. Mehen and I. W. Stewart, Nucl. Phys. A **665**, 164 (2000) [arXiv:nucl-th/9901064].

[51] D. B. Kaplan, arXiv:nucl-th/0611025.

[52] The static approximation also fails for certain observables in the single nucleon sector [42].

[53] An analogous situation has been identified in the contexts of hadronic atoms [43] and heavy quarkonium [44].